Direct Electrodisintegration and Photoeffect of Nuclei*

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A method is presented for extracting from the differential cross section for the (e,Ne') reaction the differential cross section for the corresponding (γ,N) reaction. Here N refers to a nuclear particle of spin 0 or $\frac{1}{2}$. This method is developed for arbitrary multipoles, thus extending the result given by Bosco and Fubini for $E1$ transitions. A model is used in which the ejected nuclear particle moves initially in a potential well provided by a nuclear core, and has a definite total angular momentum. The electrodisintegration process is considered to take place in such a fashion that the nuclear core is in the same state initially and finally. The electron is treated in Born approximation. In addition to the results for arbitrary multipoles, the interference of the $E1$ term with $E0$, $M1$, and $E2$ terms is studied in detail. Both the differential cross section in outgoing electron and nuclear particle directions, as well as that pertaining to the angular distribution of the latter particle alone, are given. Lastly, the case of a pure $M1$ transition is considered.

I. INTRODUCTION

THE use of inelastic-electron scattering in the study of nuclear structure is made particularly attractive by the fact that the interaction of the electron with the nucleus is purely electromagnetic, and hence well known. This same fact means that in some respects the inelastic electron-scattering process is similar to the analogous process induced by a real photon.[1] Indeed, one of two points of view may be taken in performing inelastic electron-scattering experiments. The first of these consists in using the electron in order to probe those features of nuclear structure which are necessarily inaccessible to real photons, since for electrons the momentum-transfer four vector is not a null vector, but is spacelike.[2] The second approach arises from the fact that the electron is a more convenient experimental tool than the photon, and it is desirable to perform electron experiments with a view to extracting from the data the relevant parameters in equivalent photon experiments without actually doing the latter.[3] In doing this, some of the more extensive information of the electron experiment is lost, and so this procedure is to be regarded as a matter of expediency rather than as an ultimate goal. It is the purpose of the present work to consider this second point of view in the study of the electrodisintegration problem.

Earlier work on this problem[4] has used the Born approximation and has assumed the validity of the rather stringent condition $kR\ll 1$, where k is the momentum transfer and R is the nuclear radius.[5] This second assumption causes $E1$ transitions to dominate, and the results of Bosco and Fubini are valid only for these transitions. The condition $kR\ll 1$ is not, in fact, very well satisfied. As an example, for 100 MeV electrons scattering on O^{16} the quantity kR is of order 1 for scattering through 37°. Although the majority of the electrons can be expected to scatter through an angle smaller than this,[6] it is clearly necessary to consider higher order corrections in kR to the result of Bosco and Fubini. The first-order correction will arise from the interference of $E1$ terms with $E0$, $M1$, and $E2$ terms.

For the case of pure $E1$ transitions, both the photodisintegration and the electrodisintegration cross sections are fully characterized by the same two parameters. These two parameters appear in each of the disintegration cross sections, multiplied by different known functions of the kinematics. This result has been applied to the analysis of (e,pe') data by Dodge and Barber.[7] These authors also present an expression for the electrodisintegration cross section integrated over final electron direction, thus eliminating the necessity of detecting the final electron and the ejected nucleon in coincidence.

For arbitrary multipoles with interference, it is to be anticipated that due to the transverse nature of the real photon some of the terms which appear in the electrodisintegration cross section will be absent from the photodisintegration cross section. The inelastic electron-scattering process still determines the photon process, but the converse is not necessarily true. More-

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[1] J. M. Eisenberg and M. E. Rose, Phys. Rev. **131**, 848 (1963).

[2] See, for example, Kurt Gottfried, in *Direct Interactions and Nuclear Reaction Mechanisms, Proceedings of an International Symposium, Padua, September 1962*, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, to be published).

[3] An example of this approach is the use of virtual photon spectra, see W. C. Barber, in *Annual Review of Nuclear Science*, edited by E. Segrè (Annual Reviews Inc., Palo Alto, California, 1962), Vol. 12, p. 1.

[4] B. Bosco and S. Fubini, Nuovo Cimento **9**, 350 (1958). This paper contains several printing errors which have been corrected in the paper of Dodge and Barber, Ref. 7. The relation between electrodisintegration and photodisintegration has also been considered in a paper by R. Rodenberg, Z. Physik **158**, 44 (1960). In this paper Sommerfeld-Maue wave functions are used for the electron and $E1$ and $M1$ transitions are considered. The differential cross section in outgoing nucleon is not dealt with in Rodenberg's work.

[5] We shall use units such that $\hbar=c=$ rest mass of electron $=1$.

[6] As a crude estimate, if the term dependent on the direction of the outgoing nucleon is omitted in the result of Bosco and Fubini (their $\beta=0$), then 90% of the events arise from electrons scattered through an angle $\lesssim 30°$, for 100 MeV primary energy.

[7] W. R. Dodge and W. C. Barber, Phys. Rev. **127**, 1746 (1962).

over, for arbitrary multipoles it is no longer true, as it is in the $E1$ case, that the assumption $kR \ll 1$ eliminates the dependence of the nuclear matrix element on k. This necessitates the use of some nuclear model in order to establish a result analogous to that for $E1$. In this work, a model is considered in which the ejected nucleon moves initially in a potential provided by a nuclear core, and has some fixed total angular momentum. The core is assumed to be in the same state initially and finally. The form of the nuclear current operator is left quite arbitrary, as is the form of the nuclear potential. Similar models have been used by Courant,[8] and Eichler and Weidenmüller,[9] in discussions of the $E1$ photodisintegration; these authors also make a particular choice of nuclear potential. The generality of the nuclear potential and of the nuclear current operator in the present work makes it not without meaning to consider the case of pure multipole transitions of higher order than $E1$, although no attempt is made here to discuss the details of the resonance-producing mechanism involved.[10] For pure multipole transitions a one-to-one correspondence exists between the parameters of the electron cross section and those of the photon cross section. In particular, the case of a pure $M1$ transition is studied.

The development given below applies to the case of emission of a spin-$\frac{1}{2}$ particle. The results of the very similar calculation for a spin-0 particle will be stated in an Appendix.

II. GENERAL FORMALISM

We consider the interaction between the electron and the nucleus in lowest order in the electromagnetic coupling constant and represent the electron by plane waves, so that the transition matrix element may be written in terms of Møller potentials as[11]

$$V = a_\mu \int J_\mu(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}} d\mathbf{x}, \tag{1}$$

where

$$a_\mu = -4\pi e(\bar{u}(\mathbf{p}')\gamma_\mu u(\mathbf{p}))(k^2-k_0^2)^{-1}. \tag{2}$$

Here $J_\mu(\mathbf{x})$ is the nuclear transition four-current, $(\mathbf{p},iE)$ is the four-momentum of the initial electron, $(\mathbf{p}',iE')$ is the four-momentum of the final electron, $\mathbf{k}=\mathbf{p}-\mathbf{p}'$ and $k_0=E-E'$ are the momentum and energy transfer to the nucleus, respectively. The use of the Lorentz condition for the Møller potential and the continuity equation, which is satisfied by the nuclear transition current, allows us to eliminate the fourth components of a_μ and J_μ so that we may write the matrix element as

$$V = \mathbf{a}' \cdot \int \mathbf{J}(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}} d\mathbf{x}, \tag{3}$$

where

$$\mathbf{a}' = \mathbf{a} - \mathbf{a}\cdot\mathbf{k}\mathbf{k}/k_0^2. \tag{4}$$

The differential cross section for an electron scattered into the solid-angle element $d\Omega_{e'}$ and a nucleon emitted into the solid-angle element $d\Omega_q$, assuming no polarization measurements are made, is given by

$$\frac{d^2\sigma}{d\Omega_q d\Omega_{e'}} = \frac{2\pi}{v_e} \rho_{e'}\rho_q J_{ij} N_{ij}, \tag{5}$$

where[12]

$$N_{ij} = \tfrac{1}{2} \sum_{\text{spin}} a_i'^* a_j' = \frac{(4\pi e)^2}{2EE'} \frac{1}{(k^2-k_0^2)^2} \times \Big[2p_ip_j + \tfrac{1}{2}(k^2-k_0^2)\delta_{ij} + (k_ik_j - p_ik_j - p_jk_i) \times \Big(1 - \frac{k^2}{k_0^2} + \frac{2\mathbf{p}\cdot\mathbf{k}}{k_0^2}\Big) + \frac{k_ik_j}{k_0^4} \times (2(\mathbf{p}\cdot\mathbf{k})^2 - 2k^2(\mathbf{p}\cdot\mathbf{k}) + \tfrac{1}{2}(k^2-k_0^2)k^2) \Big], \tag{6}$$

and

$$J_{ij} = \frac{1}{2j_i+1} \sum_{m_iN} \Big[\int J_i(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}} d\mathbf{x} \Big]^* \Big[\int J_j(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}} d\mathbf{x} \Big]. \tag{7}$$

In these expressions, $\rho_{e'}$ and ρ_q are the densities of states for the outgoing electron and nucleon, respectively, j_i and m_i are the initial total angular momentum and magnetic quantum numbers for the nucleon, and N is the spin projection of the nucleon in the final state. The indices i and j here refer to Cartesian coordinates. In what follows it is convenient to use spherical components, and these will be referred to with Greek letter indices.[13] In terms of these quantities, the equivalent photon cross section is given by

$$\frac{d\sigma_\gamma}{d\Omega_q} = 2\pi\rho_q J_{ij}\epsilon_i\epsilon_j, \tag{8}$$

where $\boldsymbol{\epsilon}$ is the polarization vector for the photon. If the z axis is placed along $\mathbf{k}$, this can be written, after summing over the direction of polarization, as

$$\frac{d\sigma_\gamma}{d\Omega_q} = 2\pi\rho_q(J_{ii} - J_{zz}). \tag{9}$$

[8] Ernest D. Courant, Phys. Rev. **82**, 703 (1951).

[9] J. Eichler and H. A. Weidenmüller, Z. Physik **152**, 261 (1958).

[10] Some evidence for the existence of an $E2$ resonance is observed in the experiment of Dodge and Barber, Ref. 7, who give further reference to experimental and theoretical discussion of this possibility.

[11] See, for example, Barber, Ref. 3.

[12] The spinor normalization is taken to be $(u^*u)=1$.

[13] The spherical components of a vector $\mathbf{V}$ are

$$V_\pm = \mp\sqrt{\tfrac{1}{2}}(V_x \pm iV_y)$$

and $V_0 = V_z$, so that $V_iW_i = V_\mu^* W_\mu$.

In order to establish the form of the tensor J_{ij}, we examine the spherical components of the nuclear matrix element

$$\int \mathbf{J}(\mathbf{x})\cdot\boldsymbol{\xi}_\mu \exp(i\mathbf{k}\cdot\mathbf{x})d\mathbf{x},$$

where $\boldsymbol{\xi}_\mu$ are the spherical basis unit vectors. The product of these with the plane waves is most conveniently treated by an expansion in terms of the irreducible tensors $\mathbf{T}_{\lambda l}{}^m(\hat{x})$ as given by Rose.[14] The calculation assumes its simplest form if the z axis is chosen to be along the direction of $\mathbf{k}$. In this case the expansion is

$$\boldsymbol{\xi}_\mu e^{i\mathbf{k}\cdot\mathbf{x}} = (4\pi)^{1/2}\sum_{l\lambda} i^l(2l+1)^{1/2} j_l(kx) \times C(l1\lambda;0\mu)\mathbf{T}_{\lambda l}{}^{\mu}(\hat{x}). \quad (10)$$

In terms of the model we are considering, the nuclear transition current is written as an arbitrary current operator acting between single-nucleon wave functions for the initial and final state. The initial state wave function has the form $\varphi_i(x)\chi_{j_i}{}^{m_i}(\hat{x})$, where $\chi_{j_i}{}^{m_i}(\hat{x})$ is an eigenfunction of total angular momentum and its z component with eigenvalues j_i and m_i, respectively, and $\varphi_i(x)$ is the radial wave function. The final-state wave function is expanded in terms of eigenstates of total angular momentum as

$$\psi_f{}^{N*} = 4\pi\sum_{\kappa n} i^L \zeta_L{}^*(qx) C(L\tfrac{1}{2}J; n-N, N) \times Y_L{}^{n-N}(\hat{q})\chi_\kappa{}^{n*}(\hat{x}), \quad (11)$$

where $\mathbf{q}$ is the momentum of the outgoing nucleon and $\chi_\kappa{}^n(\hat{x})$ is an eigenfunction of total angular momentum. The index κ specifies both J and L

$$\begin{aligned} J &= |\kappa| - \tfrac{1}{2}, \\ L &= \kappa, \quad \kappa > 0, \\ &= -\kappa - 1, \quad \kappa < 0. \end{aligned} \quad (12)$$

The expression for the spherical components of the nuclear matrix element is then

$$\begin{aligned} \int \mathbf{J}(\mathbf{x})\cdot\boldsymbol{\xi}_\mu &\exp(i\mathbf{k}\cdot\mathbf{x})d\mathbf{x} \\ &= (4\pi)^{3/2}\sum_{l\lambda\kappa n} i^{l+L}(2l+1)^{1/2}C(l1\lambda;0\mu) \\ &\quad\times C(L\tfrac{1}{2}J;n-N,N)Y_L{}^{n-N}(\hat{q})\int(\zeta_L{}^*(qx)\chi_\kappa{}^{n*}(\hat{x}) \\ &\quad\times \mathbf{J}^{\mathrm{op}}\varphi_i(x)\chi_{j_i}{}^{m_i}(\hat{x}))\cdot\mathbf{T}_{\lambda l}{}^{\mu}(\hat{x})j_l(kx)d\mathbf{x} \\ &= (4\pi)^{3/2}\sum_{l\lambda\kappa n} i^{l+L}(2l+1)^{1/2}C(l1\lambda;0\mu) \\ &\quad\times C(L\tfrac{1}{2}J;n-N,N)C(j_i\lambda J;m_i\mu) \\ &\quad\times \delta_{m_i+\mu,n}Y_L{}^{n-N}(\hat{q})R_{\kappa l\lambda}. \end{aligned} \quad (13)$$

The last line follows from the use of the Wigner-Eckart theorem. The reduced-matrix element is

$$R_{\kappa l\lambda} = \langle J\|(\zeta_L{}^*(qx)\mathbf{J}^{\mathrm{op}}\varphi_i(x))\cdot\mathbf{T}_{\lambda l}(\hat{x})j_l(kx)\|j_i\rangle. \quad (14)$$

From Eq. (7) we can now write the spherical components of the tensor $J_{\mu\nu}$ as

$$J_{\mu\nu} = \frac{(4\pi)^3}{2j_i+1}\sum_{l\lambda\kappa}\sum_{l'\lambda'\kappa'} i^{l+L-l'-L'}[(2l+1)(2l'+1)]^{1/2} \times R^*_{\kappa' l'\lambda'}R_{\kappa l\lambda}Q_{\mu\nu}, \quad (15)$$

where

$$\begin{aligned} Q_{\mu\nu} = \sum_{m_i N} & C(l'1\lambda';0\mu)C(l1\lambda;0\nu)C(L'\tfrac{1}{2}J';m_i+\mu-N,N) \\ &\times C(L\tfrac{1}{2}J;m_i+\nu-N,N)C(j_i\lambda'J';m_i\mu) \\ &\times C(j_i\lambda J;m_i\nu)Y_{L'}{}^{m_i+\mu-N*}(\hat{q})Y_L{}^{m_i+\nu-N}(\hat{q}). \end{aligned} \quad (16)$$

Standard recoupling procedure[14] then gives

$$\begin{aligned} Q_{\mu\nu} = (-1)^{\mu+\lambda+L'+J-J'} & (2J+1)(2J'+1)C(l'1\lambda';0\mu) \\ &\times C(l1\lambda;0\nu)\sum_\Lambda\left[\frac{(2L+1)(2L'+1)}{(2\Lambda+1)4\pi}\right]^{1/2} \\ &\times C(LL'\Lambda;00)C(\lambda\lambda'\Lambda;\nu,-\mu) \\ &\times W(\lambda j_i\Lambda J';J\lambda')W(J'\tfrac{1}{2}\Lambda L;L'J)Y_\Lambda{}^{\nu-\mu}(\hat{q}). \end{aligned} \quad (17)$$

For the electrodisintegration cross section, see Eq. (5), this tensor must be contracted against that given in Eq. (6) and so it is convenient to rewrite the latter tensor in terms of its spherical components:

$$\begin{aligned} N_{\mu\nu} = \frac{(4\pi e)^2}{2EE'}\frac{1}{(k^2-k_0{}^2)^2}\frac{4\pi}{3}\Big\{ & 2p^2Y_1{}^\mu(\hat{p})Y_1{}^{\nu*}(\hat{p}) + (k^2-k_0{}^2)\frac{3}{8\pi}\delta_{\mu\nu} + \left(1-\frac{k^2}{k_0{}^2}+\frac{2\mathbf{p}\cdot\mathbf{k}}{k_0{}^2}\right)[k^2Y_1{}^\mu(\hat{k})Y_1{}^{\nu*}(\hat{k}) \\ & - pk(Y_1{}^\mu(\hat{p})Y_1{}^{\nu*}(\hat{k}) + Y_1{}^\mu(\hat{k})Y_1{}^{\nu*}(\hat{p}))] + \frac{k^2}{k_0{}^4}[2(\mathbf{p}\cdot\mathbf{k})^2 - 2k^2(\mathbf{p}\cdot\mathbf{k}) + \tfrac{1}{2}k^2(k^2-k_0{}^2)]Y_1{}^\mu(\hat{k})Y_1{}^{\nu*}(\hat{k})\Big\}. \end{aligned} \quad (18)$$

[14] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957). The notation and conventions of this book are here used throughout.

It is thus necessary to consider five types of contractions for the electrodisintegration cross section:

$$
\begin{aligned}
Q_A &= \tfrac{1}{3}(16\pi^2)\sum_{\mu\nu} Y_1^{\mu}(\hat{k})Y_1^{\nu *}(\hat{k})Q_{\mu\nu} = 4\pi Q_{00}\,,\\
Q_B &= \tfrac{1}{3}(16\pi^2)\sum_{\mu\nu} Y_1^{\mu}(\hat{k})Y_1^{\nu *}(\hat{p})Q_{\mu\nu} = [\tfrac{1}{3}(4\pi)^3]^{1/2}\sum_{\nu} Y_1^{\nu *}(\hat{p})Q_{0\nu}\,,\\
Q_C &= \tfrac{1}{3}(16\pi^2)\sum_{\mu\nu} Y_1^{\mu}(\hat{p})Y_1^{\nu *}(\hat{k})Q_{\mu\nu} = [\tfrac{1}{3}(4\pi)^3]^{1/2}\sum_{\mu} Y_1^{\mu}(\hat{p})Q_{\mu 0}\,,\\
Q_D &= \tfrac{1}{3}(16\pi^2)\sum_{\mu\nu} Y_1^{\mu}(\hat{p})Y_1^{\nu *}(\hat{p})Q_{\mu\nu}\,,\\
Q_E &= 4\pi \sum_{\mu} Q_{\mu\mu}\,,
\end{aligned}
\tag{19}
$$

where we have used the fact that the z axis has been taken along $\mathbf{k}$. For the photodisintegration cross section summed over photon polarization direction, it is clear from Eq. (9), that only Q_A and Q_E are relevant. The results of the recoupling calculations for the quantities in Eq. (19) are

$$
\begin{aligned}
Q_A &= (-1)^{\lambda}C(l'1\lambda';00)C(l1\lambda;00)\sum_{\Lambda} K_{\lambda\lambda';\kappa\kappa'}(\Lambda)C(\lambda\lambda'\Lambda;00)P_{\Lambda}(\hat{k}\cdot\hat{q})\,,\\
Q_B &= (-1)^{l}(2\lambda+1)^{1/2}C(l'1\lambda';00)\sum_{\Lambda s} K_{\lambda\lambda';\kappa\kappa'}(\Lambda)C(l\lambda's;00)W(1l\Lambda\lambda';\lambda s)(4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{s\Lambda}{}^{0}(\hat{q})\,,\\
Q_C &= (-1)^{1-\lambda'}(2\lambda'+1)^{1/2}C(l1\lambda;00)\sum_{\Lambda s}(-1)^{\Lambda}K_{\lambda\lambda';\kappa\kappa'}(\Lambda)C(l'\lambda s;00)W(1l'\Lambda\lambda;\lambda's)(4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{s\Lambda}{}^{0}(\hat{q})\,,\\
Q_D &= (-1)^{1+l'}[(2\lambda+1)(2\lambda'+1)]^{1/2}\sum_{\Lambda\mathfrak{L}t}(-1)^{\Lambda}K_{\lambda\lambda';\kappa\kappa'}(\Lambda)C(l'lt;00)C(11\mathfrak{L};00)X(t\mathfrak{L}\Lambda;l'1\lambda';l1\lambda)\Theta_{\mathfrak{L}\Lambda t}(\hat{p},\hat{q})\,,\\
Q_E &= (-1)^{\lambda'-l'+\lambda}[(2\lambda+1)(2\lambda'+1)]^{1/2}\sum_{\Lambda} K_{\lambda\lambda';\kappa\kappa'}(\Lambda)C(ll'\Lambda;00)W(l1\Lambda\lambda';\lambda l')P_{\Lambda}(\hat{k}\cdot\hat{q})\,.
\end{aligned}
\tag{20}
$$

Here

$$
K_{\lambda\lambda';\kappa\kappa'}(\Lambda) = (-1)^{L'+J-J'}(2J+1)(2J'+1)[(2L+1)(2L'+1)]^{1/2}C(LL'\Lambda;00)W(\lambda j_i\Lambda J';J\lambda')W(J'\tfrac{1}{2}\Lambda L;L'J), \tag{21}
$$

and

$$
\Theta_{\mathfrak{L}\Lambda t}(\hat{p},\hat{q}) = 4\pi\sum_{m} C(\mathfrak{L}\Lambda t;m,-m)Y_{\mathfrak{L}}{}^{m}(\hat{p})Y_{\Lambda}{}^{-m}(\hat{q}) \tag{22}
$$

is the angular function discussed by Rose.[15] It is further convenient to define the quantity

$$
M_{\lambda\lambda';ll'}(\Lambda) = \frac{(4\pi)^2}{2j_i+1}\sum_{\kappa\kappa'} i^{L+l-L'-l'}[(2l+1)(2l'+1)]^{1/2}R^{*}_{\kappa'l'\lambda'}R_{\kappa l\lambda}K_{\lambda\lambda';\kappa\kappa'}(\Lambda)\,. \tag{23}
$$

Using Eqs. (5), (15), (18), (20), and (23), the cross section for electrodisintegration is seen to be

$$
\begin{aligned}
\frac{d^2\sigma}{d\Omega_q d\Omega_{e'}} &= \frac{2\pi}{v_e}p_{e'}p_q\frac{(4\pi e)^2}{2EE'}\frac{1}{(k^2-k_0{}^2)^2}\sum_{\Lambda}\sum_{\lambda l\lambda' l'} M_{\lambda\lambda',ll'}(\Lambda)\Big\{2p^2(-1)^{1+l'+\Lambda}[(2\lambda+1)(2\lambda'+1)]^{1/2}\sum_{\mathfrak{L}t}C(l'lt;00)C(11\mathfrak{L};00)\\
&\times X(t\mathfrak{L}\Lambda;l'1\lambda';l1\lambda)\Theta_{\mathfrak{L}\Lambda t}(\hat{p},\hat{q})+\tfrac{1}{2}(k^2-k_0{}^2)(-1)^{\lambda'-l'+l}[(2\lambda+1)(2\lambda'+1)]^{1/2}C(ll'\Lambda;00)W(l1\Lambda\lambda';\lambda l')\\
&\times P_{\Lambda}(\hat{k}\cdot\hat{q})+\left(1-\frac{k^2}{k_0{}^2}+\frac{2\mathbf{p}\cdot\mathbf{k}}{k_0{}^2}\right)[k^2(-1)^{\lambda}C(l'1\lambda';00)C(l1\lambda;00)C(\lambda\lambda'\Lambda;00)P_{\Lambda}(\hat{k}\cdot\hat{q})\\
&-pk(-1)^{l}(2\lambda+1)^{1/2}C(l'1\lambda';00)\sum_{s}C(l\lambda's;00)W(1l\Lambda\lambda';\lambda s)(4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{s\Lambda}{}^{0}(\hat{q})\\
&-pk(-1)^{1-\lambda'+\Lambda}(2\lambda'+1)^{1/2}C(l1\lambda;00)\sum_{s}C(l'\lambda s;00)W(1l'\Lambda\lambda;\lambda's)(4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{s\Lambda}{}^{0}(\hat{q})]\\
&+\frac{k^2}{k_0{}^4}[2(\mathbf{p}\cdot\mathbf{k})^2-2k^2(\mathbf{p}\cdot\mathbf{k})+\tfrac{1}{2}k^2(k^2-k_0{}^2)](-1)^{\lambda}C(l'1\lambda';00)C(l1\lambda;00)C(\lambda\lambda'\Lambda;00)P_{\Lambda}(\hat{k}\cdot\hat{q})\Big\}\,.
\end{aligned}
\tag{24}
$$

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[15] M. E. Rose, Oak Ridge National Laboratory Report ORNL-2516, 1958 (unpublished). We have taken the z axis along the direction of the third vector of these angular functions. It should also be noted that

$$(4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{s\Lambda}{}^{0}(\hat{q}) = 3^{-1/2}\Theta_{1\Lambda s}(\hat{p},\hat{q}).$$

For the analogous photodisintegration, the cross section summed over direction of photon polarization[16] is, from Eqs. (9), (15), (20), and (23),

$$\frac{d\sigma_\gamma}{d\Omega_q}=2\pi\rho_q\sum_\Lambda\sum_{\lambda l\lambda' l'} M_{\lambda\lambda';ll'}(\Lambda)\{(-1)^{\lambda'-l'+l}[(2\lambda+1)(2\lambda'+1)]^{1/2}C(ll'\Lambda;00)W(l1\Lambda\lambda';\lambda l') -(-1)^\lambda C(l'1\lambda';00)C(l1\lambda;00)C(\lambda\lambda'\Lambda;00)\}P_\Lambda(\hat{k}\cdot\hat{q}). \quad (25)$$

It should be noted that if the field in which the ejected nucleon moves in the final state is assumed negligible, that is if $\zeta_L(qx)$ is taken to be real, then the consequence of time-reversal invariance[17] is that the product of the reduced matrix elements $R^*_{\kappa' l'\lambda'}R_{\kappa l\lambda}$ is real when $L+L'+l+l'$ is even and is pure imaginary when $L+L'+l+l'$ is odd. For the emission of a spinless nuclear particle, Eqs. (24) and (25) remain valid, but the definition of the quantities in Eqs. (22) and (23) must be appropriately modified (see the Appendix).

It is the quantities $M_{\lambda\lambda';ll'}(\Lambda)$ which contain the detailed nuclear information in the problem through their dependence on the reduced matrix elements. These quantities, or combinations of them, constitute the independent parameters in the study of electrodisintegration and photodisintegration cross sections. From Eq. (10), it is seen that the contribution of a particular multipole to the cross sections in Eqs. (24) and (25) is determined by selecting the appropriate values of λ, l, λ', and l'. The case of $l=\lambda$ corresponds to an $M\lambda$ transition, while $l=\lambda-1$ gives the leading term in an $E\lambda$ transition. Terms having $l=\lambda+1$ also contribute to the $E\lambda$ transitions but are of order $(kR)^2$ compared to the $l=\lambda-1$ contribution. The $E0$ transition is present, of course, only in the electron cross section and arises from the $\lambda=0$, $l=1$ terms.[18] Thus, in the case of a pure multipole transition, with the neglect of terms of order $(kR)^2$, the quantities $\lambda=\lambda'$ and $l=l'$ have fixed values and the same independent nuclear parameters appear in both the photon and electron cross section, the number of parameters being given by the number of possible values of Λ. The triangular relations in the cross sections in Eq. (24) and (25) are such that $0\leqslant\Lambda\leqslant 2\lambda$, and the symmetry properties of the coefficients require that Λ be even, so that there are $\lambda+1$ independent parameters for an $E\lambda$ or $M\lambda$ pure transition in both the electron and photon cross sections. In the presence of interference the number of parameters appearing in the electrodisintegration case is in general larger than for photodisintegration.

III. EXAMPLES

A. Lowest Order Terms

As a first example of the use of Eqs. (24) and (25), we shall obtain expressions for the disintegration cross sections to zero and first order in kR. The zero-order expressions is arrived at by setting $l=l'=0$, see Eq. (10), which corresponds to $E1$ transitions. This gives for the electron cross section[19]

$$\frac{d^2\sigma^{E1}}{d\Omega_q d\Omega_{e'}}=\frac{2\pi}{v_e}\rho_{e'}\rho_q\frac{(4\pi e)^2}{2EE'}\frac{1}{k_0^2}\left\{\tfrac{1}{2}\alpha\left[-\frac{1}{2}+\frac{E^2+E'^2}{k^2-k_0^2}-\frac{2k_0^2}{(k^2-k_0^2)^2}\right]+\beta\left[\frac{k_0^2q^2-(\mathbf{k}\cdot\mathbf{q})^2}{2(k^2-k_0^2)}+\frac{2[(E'\mathbf{p}-E\mathbf{p}')\cdot\mathbf{q}]^2}{(k^2-k_0^2)^2}\right]\right\}, \quad (26)$$

and for the photon cross section

$$\frac{d\sigma_\gamma^{E1}}{d\Omega_\gamma}=2\pi\rho_q(\alpha+\beta\sin^2\vartheta_q), \quad (27)$$

where ϑ_q is the angle between the outgoing nucleon and the initial beam. The parameters α and β are given in our notation as

$$\alpha=-(4/\sqrt{3})M_{11;00}(0)+(\tfrac{2}{3})^{1/2}M_{11;00}(2),$$
$$\beta=-(\tfrac{3}{2})^{1/2}M_{11;00}(2). \quad (28)$$

The first-order correction in kR arises from considering terms with $l=0$, $l'=1$ and $l=1$, $l'=0$. The $l=0$, $l'=1$ case gives the interference of $E1$ with $E0$, $M1$, and $E2$ when $\lambda'=0$, 1, and 2, respectively, and similarly for the $l=1$, $l'=0$ case. The cross sections to zero plus first order in kR are given by

$$\frac{d^2\sigma}{d\Omega_q d\Omega_{e'}}=\frac{d^2\sigma^{E1}}{d\Omega_q d\Omega_{e'}}+\frac{d^2\sigma^{\rm int}}{d\Omega_q d\Omega_{e'}},$$

[16] The case in which polarization of the photon is observed may be considered by using the quantity Q_D with $\hat{p}$ replaced by $\boldsymbol{\varepsilon}$, see Eqs. (8) and (19).

[17] See S. P. Lloyd, Phys. Rev. **81**, 161 (1951).

[18] A convenient check is provided for Eq. (24) by letting $k\rightarrow k_0$ in the expression in curly brackets. From Eq. (4) it is clear that in this circumstance only transverse multipoles can contribute, and in particular the $E0$ contribution must vanish.

[19] These results have been given previously by Bosco and Fubini, Ref. 4, and by Dodge and Barber, Ref. 7.

and

$$\frac{d\sigma_\gamma}{d\Omega_q}=\frac{d\sigma_\gamma^{E1}}{d\Omega_q}+\frac{d\sigma_\gamma^{\rm int}}{d\Omega_q}.$$

The contribution to the electron cross section from the interference terms is

$$\frac{d^2\sigma^{\rm int}}{d\Omega_q d\Omega_{e'}}=\frac{2\pi}{v_e}\rho_{e'}\rho_q\frac{(4\pi e)^2}{2EE'}\frac{1}{(k^2-k_0^2)^2}\Bigg\{\mathbf{k}\cdot\hat{q}\Bigg[(\tfrac{1}{2}k^2-\tfrac{1}{2}k_0^2-\tfrac{2}{3}p^2)A-\Bigg\{k^2\left(1-\frac{k^2}{k_0^2}+\frac{2\mathbf{p}\cdot\mathbf{k}}{k_0^2}\right)$$
$$+\frac{k^2}{k_0^4}(2(\mathbf{p}\cdot\mathbf{k})^2-2k^2\mathbf{p}\cdot\mathbf{k}+\tfrac{1}{2}k^2(k^2-k_0^2))\Bigg\}B\Bigg]+\left(1-\frac{k^2}{k_0^2}+\frac{2\mathbf{p}\cdot\mathbf{k}}{k_0^2}\right)[-k^2\mathbf{p}\cdot\hat{q}C+(1/\sqrt{2})(3\mathbf{k}\cdot\hat{q}\mathbf{p}\cdot\mathbf{k}-k^2\mathbf{p}\cdot\hat{q})D]$$
$$-6^{1/2}(3\mathbf{p}\cdot\mathbf{k}\mathbf{p}\cdot\hat{q}-p^2\mathbf{k}\cdot\hat{q})E-\Bigg[(2/15)^{1/2}(5(\mathbf{p}\cdot\hat{q})^2\mathbf{k}\cdot\hat{q}-p^2\mathbf{k}\cdot\hat{q}-2\mathbf{p}\cdot\mathbf{k}\mathbf{p}\cdot\hat{q})+\tfrac{1}{2}\Bigg\{\left(1-\frac{k^2}{k_0^2}+\frac{2\mathbf{p}\cdot\mathbf{k}}{k_0^2}\right)$$
$$+\frac{1}{k_0^4}(2(\mathbf{p}\cdot\mathbf{k})^2-2k^2(\mathbf{p}\cdot\mathbf{k})+\tfrac{1}{2}k^2(k^2-k_0^2))\Bigg\}(5(\mathbf{k}\cdot\hat{q})^2-3k^2)\mathbf{k}\cdot\hat{q}+(1+\sqrt{2})\left(1-\frac{k^2}{k_0^2}+\frac{2\mathbf{p}\cdot\mathbf{k}}{k_0^2}\right)$$
$$\times\{\mathbf{p}\cdot\mathbf{k}\mathbf{k}\cdot\hat{q}-\tfrac{1}{2}\mathbf{p}\cdot\hat{q}(5(\mathbf{k}\cdot\hat{q})^2-k^2)\}\Bigg]F+(\mathbf{p}\times\hat{q})\cdot\mathbf{k}\left[2\mathbf{p}\cdot\hat{q}G+\mathbf{k}\cdot\hat{q}\left(1-\frac{k^2}{k_0^2}+\frac{2\mathbf{p}\cdot\mathbf{k}}{k_0^2}\right)H\right]\Bigg\}, \quad (29)$$

while the corresponding photon cross section is

$$\frac{d\sigma_\gamma^{\rm int}}{d\Omega_q}=2\pi\rho_q[(A+B)k\cos\vartheta_q+\tfrac{1}{2}Fk\cos\vartheta_q(5\cos^2\vartheta_q-3)]. \quad (30)$$

The various irreducible tensors arising in these expressions are to be found in Table I. The parameters may be expressed as

$$A=-(1/\sqrt{3})\sum_\lambda(2\lambda+1)^{1/2}N(1,\lambda),$$
$$B=(1/\sqrt{3})\sum_\lambda(2\lambda+1)^{1/2}(C(\lambda 11;00))^2N(1,\lambda),$$
$$C=-(1/\sqrt{3})\sum_\lambda(\delta_{\lambda 0}+(1/3)(2\lambda+1)^{1/2})N(1,\lambda),$$
$$D=\tfrac{2}{3}^{1/2}\sum_\lambda\left[(2\lambda+1)^{1/2}\frac{4}{(2-\lambda)!(\lambda+3)!}-(1/\sqrt{5})\delta_{\lambda 2}\right]\times N(1,\lambda),$$
$$E=\tfrac{1}{3}4\sqrt{2}\sum_\lambda(-1)^\lambda(2\lambda+1)^{1/2}\frac{1}{(2-\lambda)!(\lambda+3)!}N(1,\lambda),$$
$$F=-\tfrac{2}{5}^{1/2}N(3,2),$$
$$G=-i\sqrt{5}\sum_\lambda(-1)^\lambda(2\lambda+1)^{1/2}W(1121;\lambda 2)N(2,\lambda),$$
$$H=-i\sum_\lambda(\delta_{\lambda 2}+\sqrt{5}(2\lambda+1)^{1/2}W(1121;\lambda 2))N(2,\lambda), \quad (31)$$

where

$$N(\Lambda,\lambda)=2k^{-1}(4\pi)^2(2j_i+1)^{-1}\sum_{\kappa\kappa'}i^{1+L+L'}(-1)^{J-J'}\times(2J+1)(2J'+1)[(2L+1)(2L'+1)]^{1/2}\times C(LL'\Lambda;00)W(\lambda j_i\Lambda J';J1)\times W(J'1/2\Lambda L;L'J)\,\mathrm{Re}(R^*_{\kappa' 01}R_{\kappa 1\lambda}). \quad (32)$$

It is evident from Eqs. (29) and (30) that in addition to those parameters which appear both in the electrodisintegration and photodisintegration cross sections, there are also parameters which appear in the former alone. In writing the parameters in this form, it is useful to note that

$$M_{\lambda\lambda';ll'}(\Lambda)+(-1)^{l'+l+\lambda+\lambda'+\Lambda}M_{\lambda'\lambda;l'l}(\Lambda)=2(4\pi)^2(2j_i+1)^{-1}\sum_{\kappa\kappa'}i^{L+l-L'-l'}[(2l+1)(2l'+1)]^{1/2}\times K_{\lambda\lambda';\kappa\kappa'}(\Lambda)\,\mathrm{Re}(R^*_{\kappa' l'\lambda'}R_{\kappa l\lambda}), \quad (33)$$

which follows from the symmetry of the Clebsch-Gordan and Racah coefficients.[20]

The product of reduced matrix elements in Eq. (32) is linear in k to lowest order in kR, see Eq. (14), so that with the inclusion of the factor k^{-1} in Eq. (32), $N(\Lambda,\lambda)$ and all the parameters $A-H$ in Eq. (31) are independent of k to lowest order in kR. The k dependence of the interference part of the differential cross section is therefore explicit in Eq. (29) to this order. Similarly, in Eq. (26), α and β are independent of k to lowest order. The integration of Eq. (26) and Eq. (29) over final electron direction is then carried out by transforming variables from $\cos\vartheta$, where ϑ is the angle between $\mathbf{p}$ and $\mathbf{p}'$, to k^2. The integrals which appear are of the form

$$I_n=\int_{(p-p')^2}^{(p+p')^2}\frac{k^{2n}dk^2}{(k^2-k_0^2)^2}, \quad (34)$$

[20] In accord with the comment made below Eq. (25), it is seen that G and H of Eq. (31) vanish in the absence of a final-state interaction for the nucleon. This is so for G and H, because for these quantities $\Lambda=2$, and thus due to the presence of the parity Clebsch-Gordan coefficient $C(LL'\Lambda;00)$ in Eq. (21), $L+L'$ = even and $L+L'+l+l'=L+L'+1$ = odd. The product $R^*_{\kappa' 01}R_{\kappa 1\lambda}$ is, therefore, pure imaginary.

TABLE I. Irreducible tensors which are used in the calculation.[a] The arguments of the functions are understood: $\Theta_{\mathcal{L}\Lambda t}(\hat{p},\hat{q})$, $\mathbf{T}_{s\Lambda}{}^0(\hat{q})$, with the z axis along $\mathbf{k}$.

$$\Theta_{\Lambda\Lambda 0}=(2\Lambda+1)^{1/2}P_\Lambda(\hat{p}\cdot\hat{q}) \qquad \Theta_{211}=-\tfrac{3}{2}^{1/2}(3\hat{p}\cdot\hat{k}\hat{p}\cdot\hat{q}-\hat{k}\cdot\hat{q})$$

$$\Theta_{011}=3^{1/2}\hat{k}\cdot\hat{q} \qquad \Theta_{221}=(15i/10^{1/2})\hat{p}\cdot\hat{q}\hat{q}\times\hat{p}\cdot\hat{k}$$

$$\Theta_{231}=\tfrac{3}{2}[5(\hat{k}\cdot\hat{q})(\hat{p}\cdot\hat{q})^2-\hat{k}\cdot\hat{q}-2\hat{p}\cdot\hat{k}\hat{p}\cdot\hat{q}]$$

$$\Theta_{022}=(5^{1/2}/2)(3(\hat{k}\cdot\hat{q})^2-1) \qquad \Theta_{202}=(5^{1/2}/2)(3(\hat{k}\cdot\hat{p})^2-1)$$

$$\Theta_{222}=(5/14^{1/2})(1-3\hat{p}\cdot\hat{q}\hat{p}\cdot\hat{k}\hat{k}\cdot\hat{q}-3((\hat{q}\times\hat{p})\cdot\hat{k})^2)$$

$$(4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{01}{}^0=-\hat{p}\cdot\hat{q} \qquad (4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{10}{}^0=\hat{p}\cdot\hat{k}$$

$$(4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{12}{}^0=2^{-1/2}(\hat{p}\cdot\hat{k}-3\hat{k}\cdot\hat{q}\hat{p}\cdot\hat{q}) \qquad (4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{21}{}^0=2^{-1/2}(3\hat{k}\cdot\hat{q}\hat{p}\cdot\hat{k}-\hat{p}\cdot\hat{q})$$

$$(4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{22}{}^0=-i(15/2)^{1/2}\hat{k}\cdot\hat{q}(\hat{p}\times\hat{q}\cdot\hat{k}) \qquad (4\pi)^{1/2}\hat{p}\cdot\mathbf{T}_{23}{}^0=(3^{1/2}/2)(2\hat{k}\cdot\hat{q}\hat{p}\cdot\hat{k}-\hat{p}\cdot\hat{q}(5(\hat{k}\cdot\hat{q})^2-1))$$

[a] The following vector identity is useful in evaluating these tensors: $(\hat{q}\times\hat{p}\cdot\hat{k})^2+(\hat{p}\cdot\hat{k})^2+(\hat{p}\cdot\hat{q})^2+(\hat{k}\cdot\hat{q})^2-2\hat{p}\cdot\hat{k}\hat{k}\cdot\hat{q}\hat{p}\cdot\hat{q}=1$.

which can be generated from the recursion relations

$$k_0{}^4I_n-2k_0{}^2I_{n+1}+I_{n+2}=(n+1)^{-1}[(p+p')^{2n+2}-(p-p')^{2n+2}]. \quad (35a)$$

The first two integrals are

$$I_0=pp'/k_0{}^2, \quad (35b)$$

and

$$I_1=2\lambda+pp', \quad (35c)$$

where

$$\lambda=\ln[(EE'+pp'-1)/k_0]. \quad (35d)$$

For the pure $E1$ transitions the integration gives[21]

$$\frac{d\sigma^{E1}}{d\Omega_q}=\frac{2\pi^2}{v_e}\rho_{e'}\rho_q\frac{(4\pi e)^2}{2EE'}\frac{1}{k_0{}^2}\bigg\{\alpha\bigg[\frac{E^2+E'^2}{pp'}\lambda-2\bigg]$$

$$+\frac{\beta}{2pp'}[\sin^2\vartheta_q\{\lambda[2k_0{}^2-3p'^2+(3/p^2)(EE'-1)^2+(12E^2/p^2)(EE'-1)-8EE']-(p'/p)(4E^2+3EE'+5)\}$$

$$+2\lambda\{p'^2-(1/p^2)(EE'-1)^2-(4E^2/p^2)(EE'-1)+4EE'\}+(2p'/p)(3+EE')]\bigg\}. \quad (36)$$

In the electron case, ϑ_q is the angle between the direction of the outgoing nucleon and the initial electron beam. For the interference contribution, the cross section is best left in terms of the integrals defined by Eq. (34). It is also convenient to introduce the quantity

$$y=p^2-p'^2=k_0(E+E'). \quad (37)$$

The interference contribution is

$$\frac{d\sigma^{\text{int}}}{d\Omega_q}=\frac{2\pi^2}{v_e}\rho_{e'}\rho_q\frac{(4\pi e)^2}{2EE'}\frac{1}{2pp'}\cos\vartheta_q\bigg\{(A/2p)[yI_1+I_2-(yI_0+I_1)(k_0{}^2+4p^2/3)]-(B/p)\bigg[(yI_1+I_2)\bigg(1+\frac{y}{k_0{}^2}+\frac{y^2}{2k_0{}^4}\bigg)$$

$$-\frac{1}{2k_0{}^2}(yI_2+I_3)\bigg]-2CpI_1\bigg(1+\frac{y}{k_0{}^2}\bigg)+\frac{D}{\sqrt{2}}\bigg(1+\frac{y}{k_0{}^2}\bigg)\bigg[\frac{3}{2p}(y^2I_0+2yI_1+I_2)-2pI_1\bigg]-2(6)^{1/2}Ep(yI_0+I_1)$$

$$-F\bigg[(2/15)^{1/2}p(yI_0+I_1)(5\cos^2\vartheta_q-3)+(5/16p^3)(5\cos^2\vartheta_q-3)(y^3I_0+3y^2I_1+3yI_2+I_3)\bigg(1+\frac{y}{k_0{}^2}+\frac{y^2}{2k_0{}^4}\bigg)$$

$$+(15/4p)\sin^2\vartheta_q(yI_1+I_2)\bigg(1+\frac{y}{k_0{}^2}+\frac{y^2}{2k_0{}^4}\bigg)-(5/32k_0{}^2p^3)(5\cos^2\vartheta_q-3)(y^3I_1+3y^2I_2+3yI_3+I_4)$$

$$-(15/8k_0{}^2p)\sin^2\vartheta_q(yI_2+I_3)-(3/2p)(yI_1+I_2)\bigg(1+\frac{y}{k_0{}^2}+\frac{y^2}{2k_0{}^4}\bigg)$$

$$+(3/4k_0{}^2p)(yI_2+I_3)-\frac{1+\sqrt{2}}{2}p\bigg(1+\frac{y}{k_0{}^2}\bigg)(5\cos^2\vartheta_q-3)\{(3/4p^2)(y^2I_0+2yI_1+I_2)-I_1\}\bigg]\bigg\}. \quad (38)$$

[21] For E, $E'\gg1$, this result has been given previously by Dodge and Barber, Ref. 7.

To zero plus first order in kR, the differential cross section in outgoing nucleon direction is given by the sum of Eqs. (36) and (38).

B. Pure $M1$ Transitions

In the case of a pure magnetic transition, due to the transverse nature of the magnetic multipole, the quantities Q_A, Q_B, and Q_C of Eqs. (19) and (20) vanish ($C(\lambda 1\lambda;\,00)=0$). For an $M1$ transition the cross section for electrodisintegration is, from Eq. (24),

$$\frac{d^2\sigma^{M1}}{d\Omega_q d\Omega_{e'}}=\frac{2\pi}{v_e}\rho_{e'}\rho_q\frac{(4\pi e)^2}{2EE'}\frac{1}{(k^2-k_0^2)^2}\{a[k^2p^2-(\mathbf{k}\cdot\mathbf{p})^2+\tfrac{1}{2}k^2(k^2-k_0^2)]+b[k^2(\mathbf{p}\cdot\hat{q})^2+p^2(\mathbf{k}\cdot\hat{q})^2-2(\hat{q}\times\mathbf{p}\cdot\mathbf{k})^2-2\mathbf{p}\cdot\hat{q}\mathbf{p}\cdot\mathbf{k}\mathbf{k}\cdot\hat{q}+\tfrac{1}{4}(k^2-k_0^2)(3(\mathbf{k}\cdot\hat{q})^2-k^2)]\}. \quad (39)$$

The photodisintegration cross section is

$$\frac{d\sigma_\gamma^{M1}}{d\Omega_q}=6\pi\rho_q[ak^2+b(3(\mathbf{k}\cdot\hat{q})^2-k^2)]. \quad (40)$$

The relevant irreducible tensors appear in Table I. The parameters in Eqs. (39) and (40) are

$$a=(1/\sqrt{3}k^2)M_{11;11}(0),$$
$$b=(1/\sqrt{6}k^2)M_{11;11}(2).$$

These two quantities are independent of k to order $(kR)^2$. The integration over final electron direction is easily carried out as in Sec. IIIA and yields

$$\frac{d\sigma^{M1}}{d\Omega_q}=\frac{2\pi^2}{v_e}\rho_{e'}\rho_q\frac{(4\pi e)^2}{2EE'}\frac{1}{2pp'}\{2a\lambda(p^2+p'^2)+b(1-3\cos^2\vartheta_q)\times[2\lambda(1-EE')+2pp'-(k_0^2/2p^2)\lambda(2E^2+1)+(p'/2p)(-4E^2+3EE'+1)]\}. \quad (42)$$

In this pure multipole transition, the same two parameters, a and b, appear in both the photodisintegration and electrodisintegration cross sections. It will be observed that if the integration over emitted nucleon direction is carried out, then both the photon and the electron cross sections involve only the parameter a and hence are proportional to each other. Moreover, the proportionality factor is a known function of the kinematics.[22] In the extreme relativistic limit, $E, E'\gg 1$:

$$\frac{d\sigma^{M1}}{d\Omega_q}=\frac{2\pi^2}{v_e}\rho_{e'}\rho_q\frac{(4\pi e)^2}{2EE'}\left[a\frac{E^2+E'^2}{EE'}\lambda+\frac{b}{2}(1-3\cos^2\vartheta_q)\left(\frac{3}{2}\frac{E'}{E}-\frac{E^2+E'^2}{EE'}\lambda\right)\right]. \quad (43)$$

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APPENDIX

In this Appendix the results will be stated for the case in which the ejected nuclear particle is spinless. The calculation proceeds as in Sec. II, except that the final nuclear particle wave function [corresponding to Eq. (11)] is taken to be

$$\psi_f^*=4\pi\sum_{LM} i^L\zeta_L^*(qx)Y_L^M(\hat{q})Y_L^{M*}(\hat{x}). \quad (A1)$$

The results for the electrodisintegration and photodisintegration cross sections are given in Eqs. (24) and (25), respectively, provided that in these equations the following replacement is made:

$$M_{\lambda\lambda';ll'}(\Lambda)\rightarrow\bar{M}_{\lambda\lambda';ll'}(\Lambda)=\frac{(4\pi)^2}{2l_i+1}\sum_{LL'} i^{L+l-L'-l'}[(2l+1)(2l'+1)]^{1/2}\times R^*_{L'l'\lambda'}R_{Ll\lambda}\bar{K}_{\lambda\lambda';LL'}(\Lambda), \quad (A2)$$

with

$$\bar{K}_{\lambda\lambda';LL'}(\Lambda)=(-1)^L(2L+1)(2L'+1)\times C(LL'\Lambda;\,00)W(\lambda l_i\Lambda L';\,L\lambda'). \quad (A3)$$

Here l_i is the initial orbital angular momentum and $R_{Ll\lambda}$ is the reduced matrix element. The remaining quantities are as defined in Sec. II.

[22] This is the starting point for Rosenberg's calculation (Ref. 4).